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Quark Pair Creation, Electromagnetic Masses and G-Violating ω, η Decays

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ABSTRACT

The quark pair creation model, recently found to simulate without parameters the Coleman-Glashow mechanism for e.m. masses in remarkable accord with experiment, is shown to provide rather good fits to $\eta \to 3\pi$ and $\omega \to 2\pi$ decays in a very simple way.

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Physical interest in the $\eta \to 3\pi$ problem has been sustained by the fact that while the violation of G-parity is believed to stem from e.m. effects,[1] even the most optimistic estimates seem to fall far short of the observed magnitude.[2] More quantitatively, the role of the Coleman-Glashow tadpole[3] was recognized quite early in this context,[4] but the main problem has been one of generating in a sufficiently convincing manner an adequate strength of the effect within the generally accepted framework of chiral symmetry[5] breaking theories[6, 7], including more sophisticated treatments in recent times.[8, 9] The closely allied problem of $\omega \to 2\pi$ decay (which does not seem to have received a comparable degree of attention) should presumably be entitled to a similar treatment. Indeed the two processes could conceivably serve as mutual checks on the credibility of any concrete theory that claims to understand at least one of them. The purpose of this note is to describe one such mechanism which, to our surprise, seems to account simultaneously for both phenomena in a rather natural way, yielding numbers--270 KeV and 215 eV for $\omega \to 2\pi$ and $\eta \to 3\pi$ respectively--in very good accord with experiment, with only the $\rho \to \pi\pi$ width as input.

The model proposed is based on the quark-pair creation (QPC) hypothesis for the construction of hadronic vertices[10] whose strengths are proportional to the overlap integrals of the normalized harmonic oscillator(h. o.) wave functions ($q\bar{q}$ or qqq) of the concerned hadrons, with the resultant form factors given relativistic meaning.[11] It was recently applied to the problem of e.m. masses of hadrons[12] with a combination of tadpole[3] and Born contributions, with excellent fits to all the octet mass differences. The tadpole contribution is simulated in this model through an entire sequence of A_J (J-even) mesons, as shown in Fig. 1(c). The summation over different A_J contributions turns out to be dominated by a fixed pole at J = -2 in the Watson-Sommerfeld background integral (not Regge pole!) and yields an explicit formula for δm or δm^2 in terms of the hadron mass involved. While the possibility of such a fixed pole has been recognized in the literature[13], its effect on e.m. masses does not seem to have been worked out explicitly. In our model, the $K^0 - K^+$ $(\text{mass})^2$ difference is given by the formula:

$$\delta m_K^2 = \frac{G^2 e^2}{g_\rho^2} \frac{27\pi^{3/2}}{(2\pi)^5} \frac{\exp\left(\frac{2}{3} m_K^2\right)}{m_K^2 + m_\rho^2} \approx 0.006 \text{ GeV}^2 \tag{1}$$

This tadpole formula for kaons should in principle be valid also for "transition masses", such as those appearing in $\omega\rho^0$ and $\eta\pi^0$ couplings via the A_2-tadpole, provided one uses the actual masses of the hadrons involved (ω or η in the present case) instead of the physical kaon mass. This point of view represents our ansatz for the construction of the effective strengths of $\omega\rho^0$ and $\eta\pi^0$ couplings via electromagnetic masses, for determining their respective rates of G-parity violating decays through Figs. 1(a, b).

We define the effective Lagrangians for $\omega\rho^0$ and $(\eta, \eta')\pi^0$ couplings as

$$\mathscr{L}_\omega = \delta m_\omega^{\,2} \omega_\mu \rho_\mu^{\,0}; \quad \mathscr{L}_{\eta,\,\eta'} = \frac{1}{\sqrt{2}}\, \delta m_{\eta,\,\eta'}^{\,2} \pi^0 (\eta, \eta') \tag{2}$$

where $\delta m_\omega^{\,2}$, etc. are formally expressed by Eq. (1) but with the replacements $m_K^{\,2} \to m_\omega^{\,2}$, etc. Eq. (2) incorporates necessary adjustments in the SU(6) factors involved in $K\overline{K}$ versus $\omega\rho$ and $(\eta, \eta')\pi$ couplings to A_2. The factors $\frac{1}{\sqrt{2}}$ in the η, η' cases are in conformity with the usual $\eta_1\eta_8$ mixing angle[14], but with the states defined in an "ideal" basis.[15]

The formula (1) is strictly independent of the strong interaction coupling constant G, representing the proportionality factor in the spatial overlap integrals for the various hadron vertices[10], since the rho coupling constant g_ρ in this model is porportional to G[12]:

$$g_\rho = 2m_\rho G\left(\frac{4}{9\pi}\right)^{3/4} \exp\left(\frac{2}{3}\, m_\rho^{\,2}\ \mathrm{GeV}^{-2}\right) \tag{3}$$

For the calculation of ω, η widths, however, a dependence on G, or g_ρ, comes from the non-tadpole vertices involved in the matrix elements (Fig. 1), and we normalize this quantity to the value

$$g_\rho^{\,2}/4\pi = 2.5 \tag{4}$$

to yield a $\rho \to \pi\pi$ width of 150 MeV.

$\omega \to 2\pi$ Decay:

The ρ^0-dominance, Fig. 1(a), for the process allows only a $\pi^+\pi^-$ mode, in conformity with experiment.[14] The ρ^0-propagator in the matrix element is prevented from vanishing ($m_\rho^2 - m_\omega^2 \approx -m_\pi^2$)by the replacement $m_\rho^2 \to m_\rho^2 - i\, m_\rho \Gamma_\rho$, where we take[14]

$$m_\rho \Gamma_\rho \approx 0.12 \text{ GeV}^2 \quad . \tag{5}$$

The resulting formula gives

$$\Gamma_{\omega \to 2\pi} \approx \left(\frac{\delta m_\omega^2}{\Gamma_\rho m_\rho}\right)^2 \frac{g_\rho^2}{4\pi} \left(\frac{k}{6}\right) \approx 270 \text{ KeV}, \tag{6}$$

to be compared with a measured value[14] of about 200 KeV (within errors). In achieving this near agreement, the effect of the ρ-width has been decisive, but for which the figure would have been astronomical ($\sim$ 10 MeV!) The m_ω versus m_K effect in Eq. (1) has also brought about a useful 30% reduction.

$\eta \to 3\pi$ Decay:

This process, Fig. 1(b), could in principle be mediated by $\pi^0\rho^\pm\pi^\mp$ and/or $\pi^0\epsilon\pi^0$ vertices where ϵ is the so-called scalar meson (I = 0) supposed to be about as massive as the rho. However, on the basis of energy and angular distributions of the decay pions in $\eta \to 3\pi$ found many years ago[16], and confirmed by continued data evidence[14], one would expect a dominance of s-waves in the pion pairs, thus favoring $\underline{\epsilon}$ over $\underline{\rho}$. Indeed the $\eta \to 3\pi$ analysis[16] had appeared to support certain s-wave π-π resonance proposals in the early sixties[17] (though the mass and width estimates had been wide off the $\underline{\epsilon}$-mark). We also suspect that the success of the SU(3) $\times$ SU(3) σ-model in explaining the energy spectrum of the decay pions[7] is in good measure due to strong s-wave π-π effects implied in such a model. Further, the ratio of $3\pi^0$ to $\pi^+\pi^-\pi^0$ modes in an $\underline{\epsilon}$-dominance model would be given by a statistical-cum-isospin factor of 3/2, computed from Bose statistics[16], except for corrections due to the finite mass of $\underline{\epsilon}$. A pure ρ-dominance would give <u>zero</u> for this ratio, but a correction due to ρ-effects would tend to <u>decrease</u> this value, in conformity with the data.[14]

Despite these attractive features of $\underline{\epsilon}$, a calculation of $\eta \to 3\pi$ via the ϵ-meson would not be of physical interest, if it amounted to the introduction of a fresh coupling constant for the sake of a single experimental parameter. Fortunately, within the framework of our model, we <u>can</u> calculate <u>both</u> ρ- and ϵ-mediated matrix elements without any free parameters, so that their relative contributions to the $\eta \to 3\pi$ width can be explicitly evaluated and compared. The $\epsilon\pi\pi$ coupling is dominated by the so-called quark-recoil effect which, in the QPC model[10], comes without a free parameter (the quark mass), as was stressed recently in connection with an explanation of the reduced widths observed in $V \to P\gamma$ decays.[18] In the present case, the "direct" term ($\vec{\sigma} \cdot \vec{k}$) contribution to $\epsilon\pi\pi$ coupling is a small ($\lesssim 5\%$)

effect, so that using the "recoil" term alone, the (dimensional) $\epsilon\pi\pi$ coupling constant g_ϵ is expressible in terms of g_ρ as

$$g_\rho = 2m_\rho \sqrt{3}\, g_\epsilon \tag{7}$$

With these ingredients we have calculated both the ρ- and ϵ-contributions to the charged pion decay of η, on the assumption of equal ρ and ϵ masses, and obtained 15 eV and 200 eV respectively, to be compared with the latest experimental value[14] of 210 ± 30 eV. The explicit formulae are omitted for brevity, but the main reason for the small ρ-contribution is the barrier factor associated with p-wave effects in a relatively small phase space. The ϵ-dominance automatically ensures the correct neutral to charged pion ratio, as anticipated in the foregoing.

For completeness we record the results for $\eta' \rightarrow 3\pi$ decay, though for this case the data are not clear enough.[14] Since the $\rho\pi$ and $\epsilon\pi$ channels are now physically accessible, we assume that the resultant charged pion modes are saturated by these two channels which yield the figures of 430 eV and 370 eV respectively. Since the ρ-contribution is now somewhat higher than ϵ, one should expect to see appreciable p-wave effects in the pion angular and energy distributions in $\eta' \rightarrow 3\pi$ decay, unlike the $\eta \rightarrow 3\pi$ case.

This investigation arose out of a suggestion from Prof. L. K. Pandit and Prof. P. P. Divakarau to check the success of the tadpole mechanism in the QPC model against some related decay processes. I am grateful to Prof. Y. Nambu for a most valuable suggestion, viz., the inclusion of the ρ-width in the $\omega \rightarrow \pi\pi$ process, and to Prof. L. Balazs for useful comments. Finally, it is a pleasure to acknowledge the hospitality of Fermilab where this work was carried out, especially to Prof. B. W. Lee, Prof. C. Quigg and the Director, Prof. R. R. Wilson.

REFERENCES

[1] For an early review, see, e.g., J. S. Bell and D. Sutherland, Nucl. Phys. B4, 315 (1968).

[2] G. Cicogna, et al., Lett. Nuovo Cimento 10, 25 (1974).

[3] S. Coleman and S. L. Glashow, Phys. Rev. 134, B 671 (1964).

[4] S. K. Bose and A. Zimerman, Nuovo Cimento 43A, 1165 (1966).

[5] M. Gell Mann, R. J. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968).

[6] N. Cabibbo and L. Maiani, Phys. Rev. D1, 707 (1970).

[7] W. Hudnall and J. Schechter, Phys. Rev. D9, 2111 (1974).

[8] S. Weinberg, Phys. Rev. D11, 3583 (1975).

[9] A. Golestaneh, FERMILAB-Pub-76/18-THY,1976; also for other references.

[10] A. Le Yaouanc et al., Phys. Rev. D8, 2223 (1973); Y. S. Kim and M. Noz, ibid. D12, 122 (1975).

[11] A. N. Mitra, Proc. of the Intl. Workshop on Quarks and Hadronic Structure, Erice, Sept. 1975; Ed. G. Morpurgo, in press.

[12] A. N. Mitra, Phys. Lett. (1976); in press.

[13] E. G. D. Dicus and V. Teplitz, Phys. Rev. D4, 2313 (1971); A. Zee, Phys. Report 3C, 128 (1972).

[14] Particle Data Group. Rev. Mod. Phys. 48, S1 (1976).

[15] A. Bramon and M. Greco, Phys. Rev. Lett. 48B, 373 (1974).

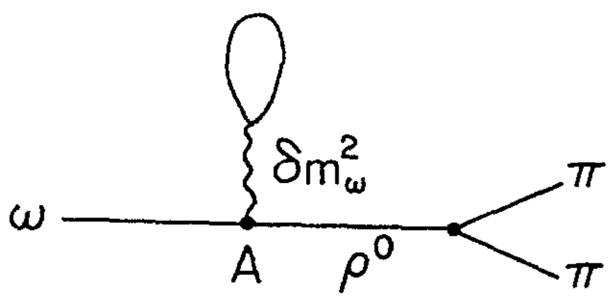
[16] F. Crawford et al., Phys. Rev. Lett. 11, 564 (1963).

[17] F. Brown and P. Singer, Phys. Rev. Lett. 8, 460 (1962); A. N. Mitra, in the Proc. of the X Intl. Conf. on HEP, Rochester 1960).

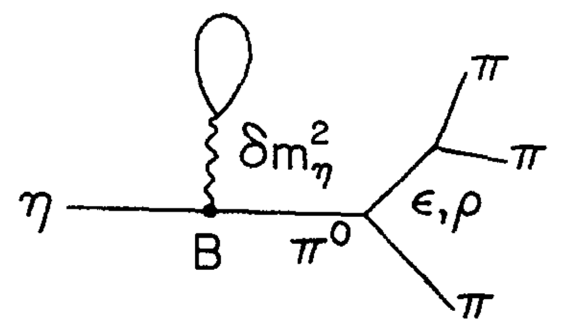
[18] A. N. Mitra, Phys. Rev. D13, (1976), in press.

FIGURE CAPTION

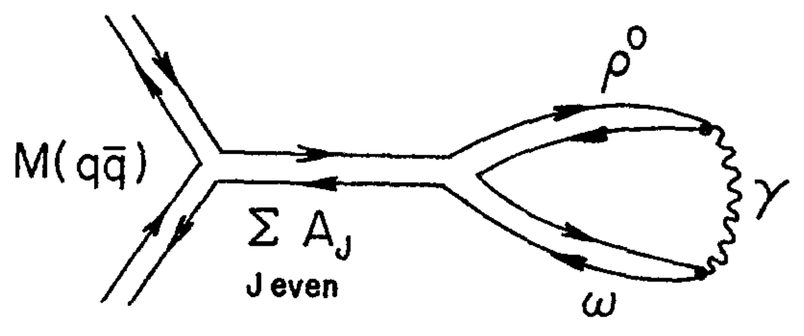
Fig. 1 (a, b): Tadpole mechanisms for $\omega \to 2\pi$ and $\eta \to 3\pi$ decays. The $q\bar{q}$ structure of the δm^2 vertices at A and B is shown in Fig. 1(c).


δm^2_ω
π
ω
A
ρ^0
π

(a)


π
δm^2_η
π
η
ϵ, ρ
B
π^0
π

(b)


ρ^0
$M(q\bar{q})$
γ
$\Sigma\ A_J$
J even
ω

(c)